Local limit of Mallows trees

TU/e

Benoît Corsini

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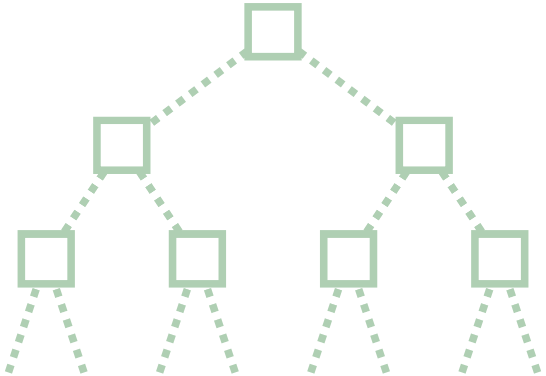

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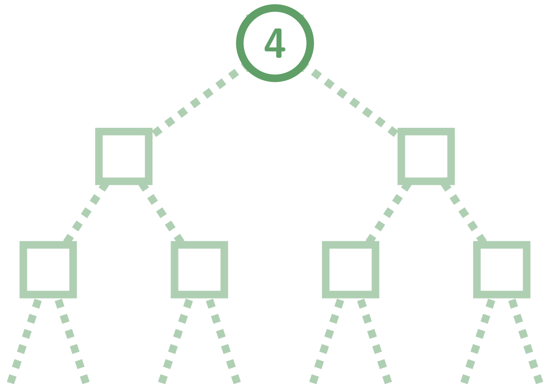

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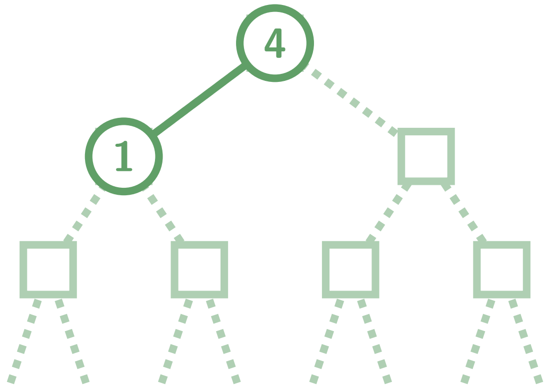

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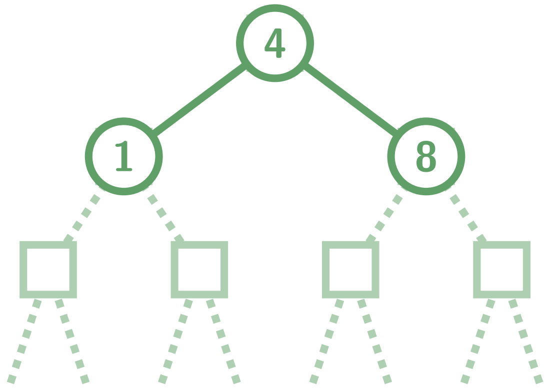

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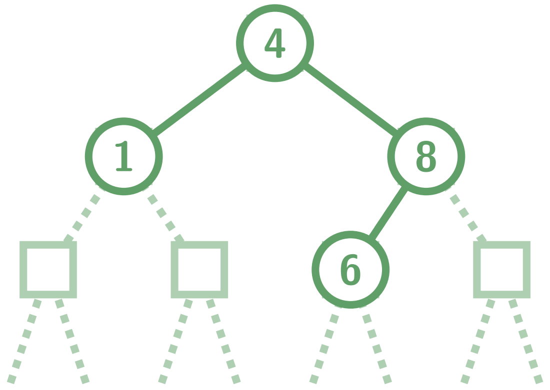

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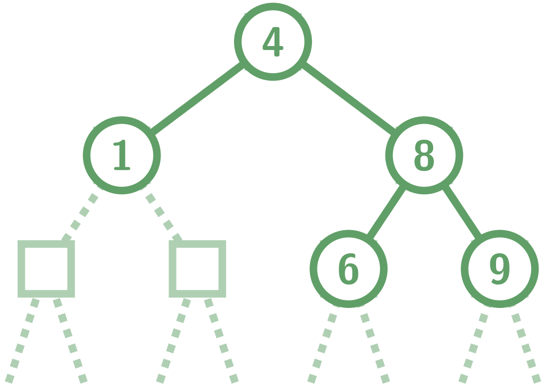

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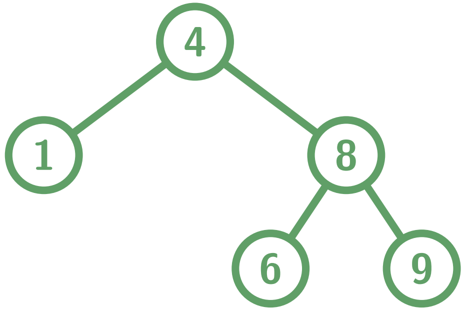

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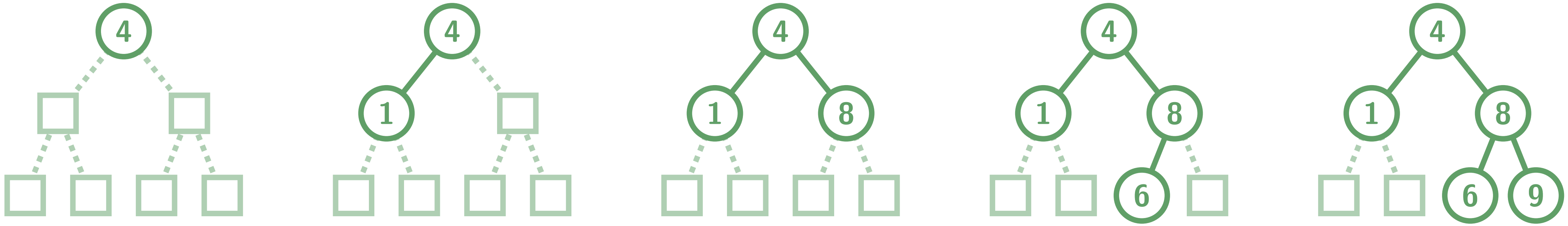

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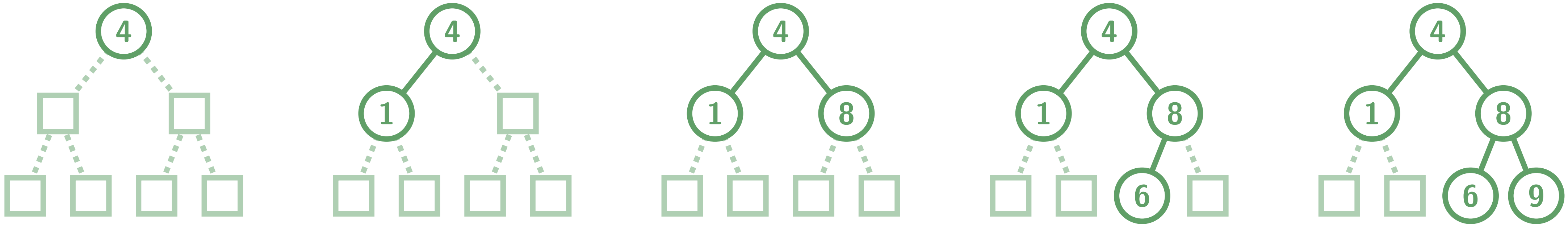

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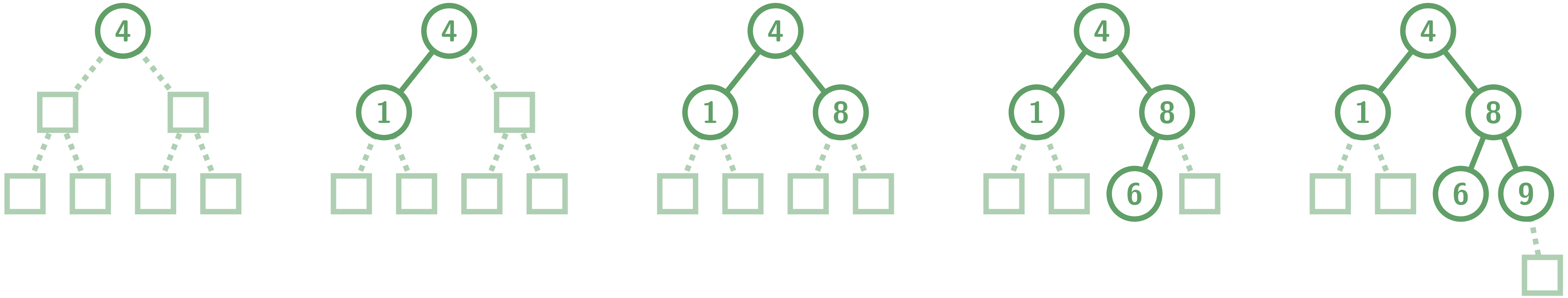

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Mallows permutations

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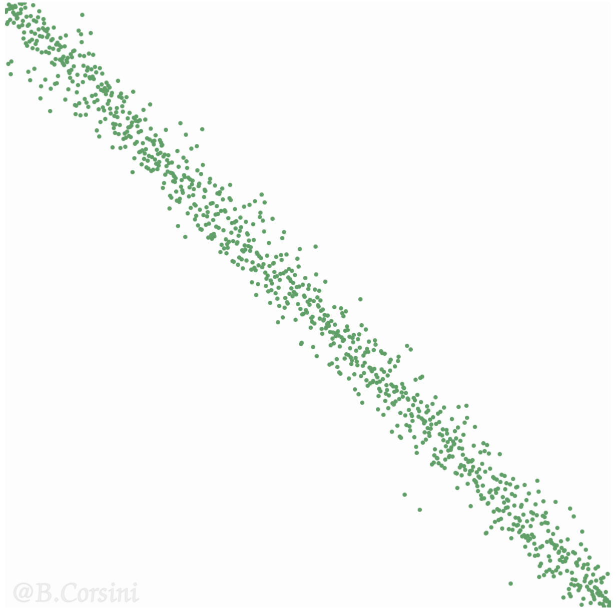

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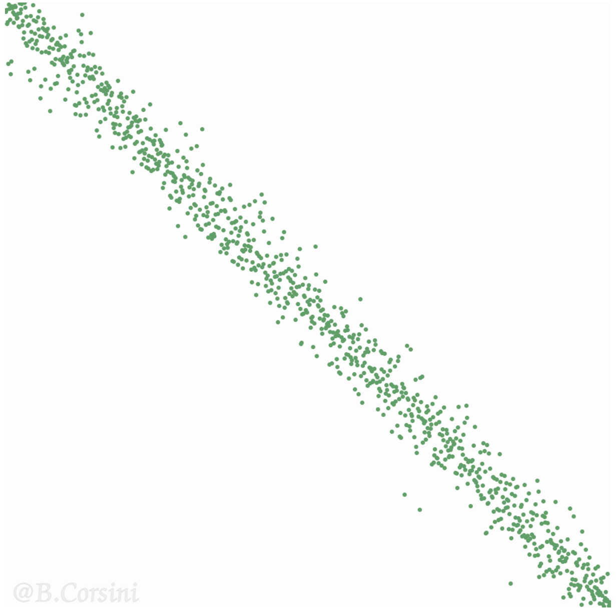

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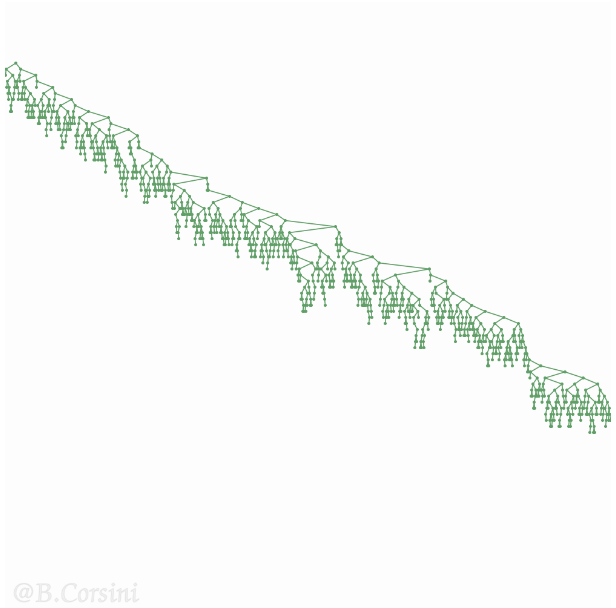

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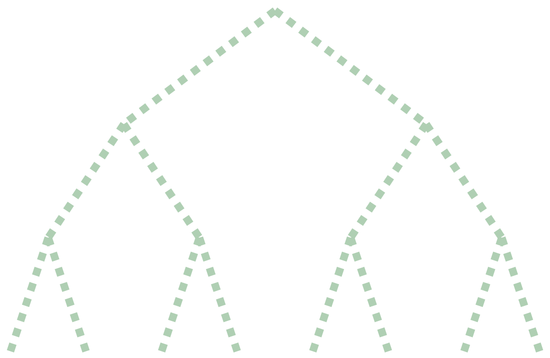

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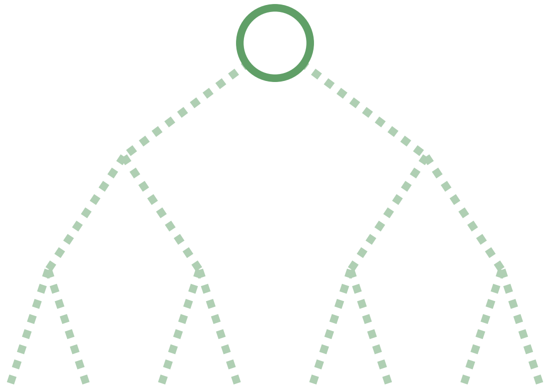

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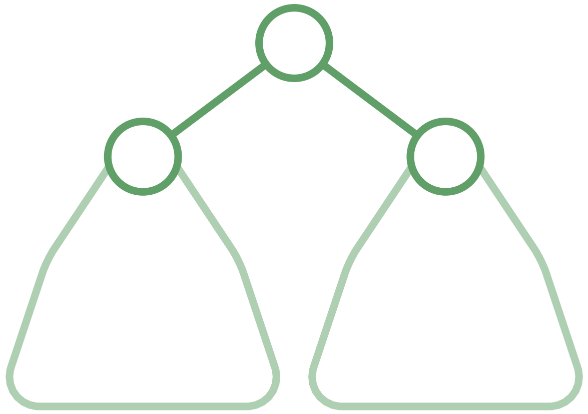

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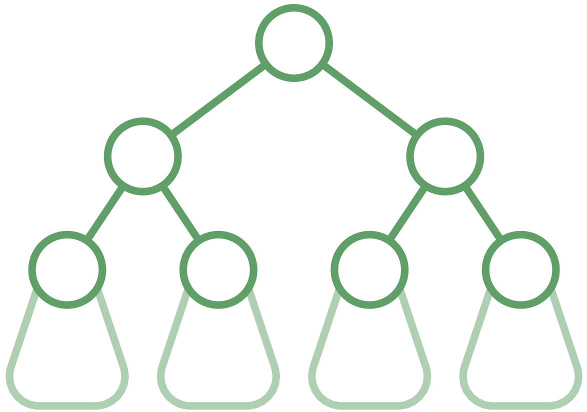

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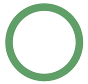

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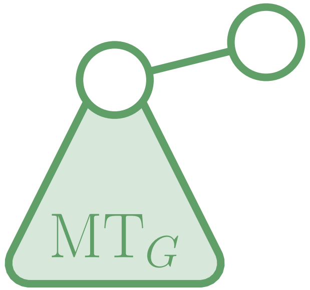

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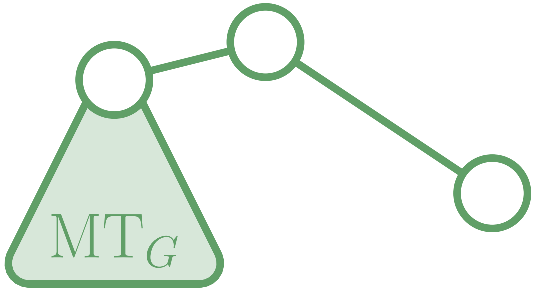

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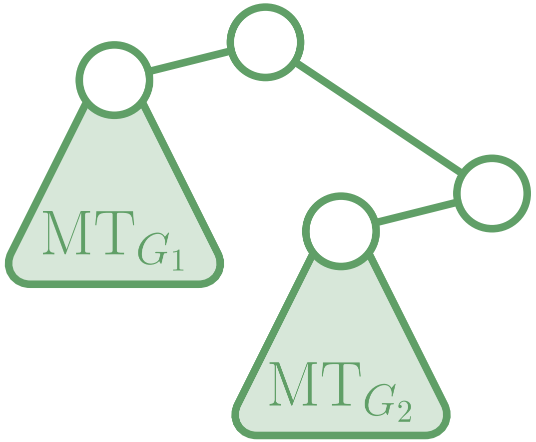

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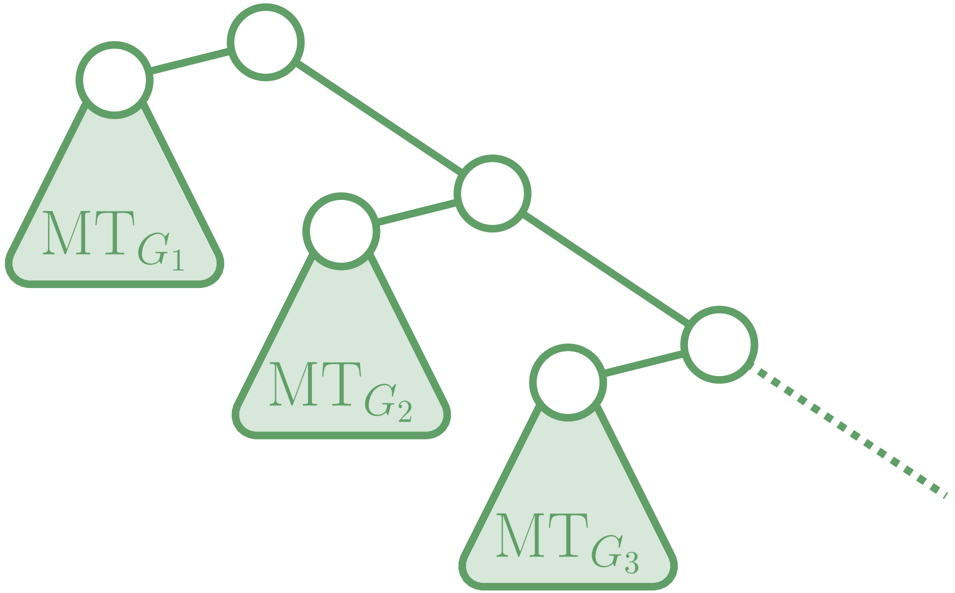

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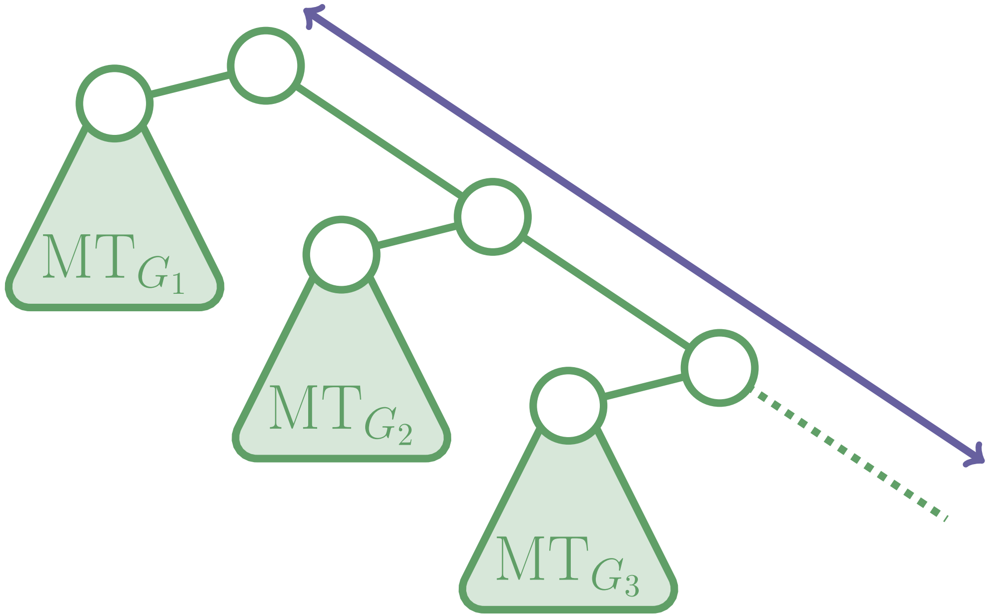

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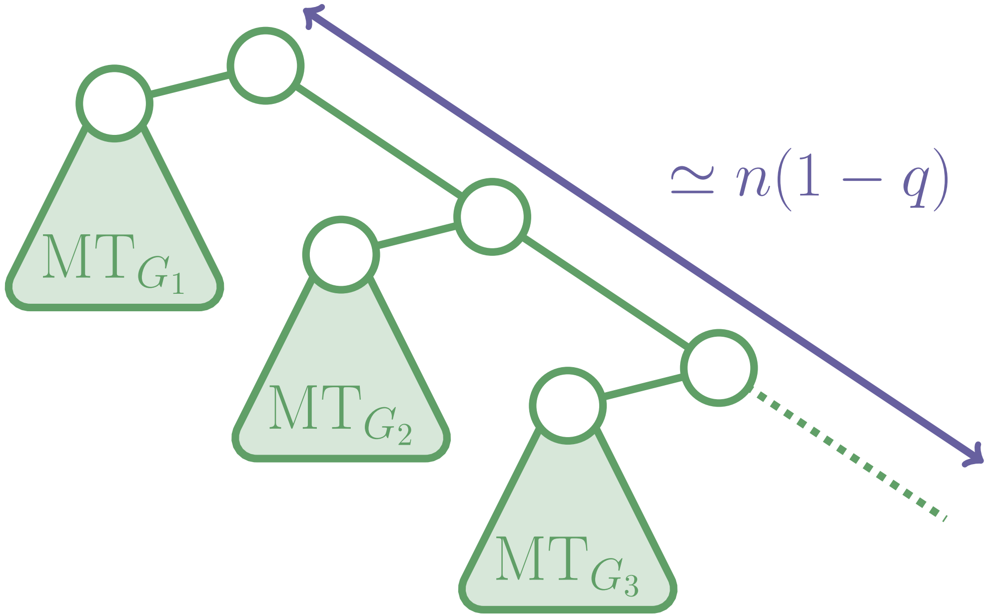

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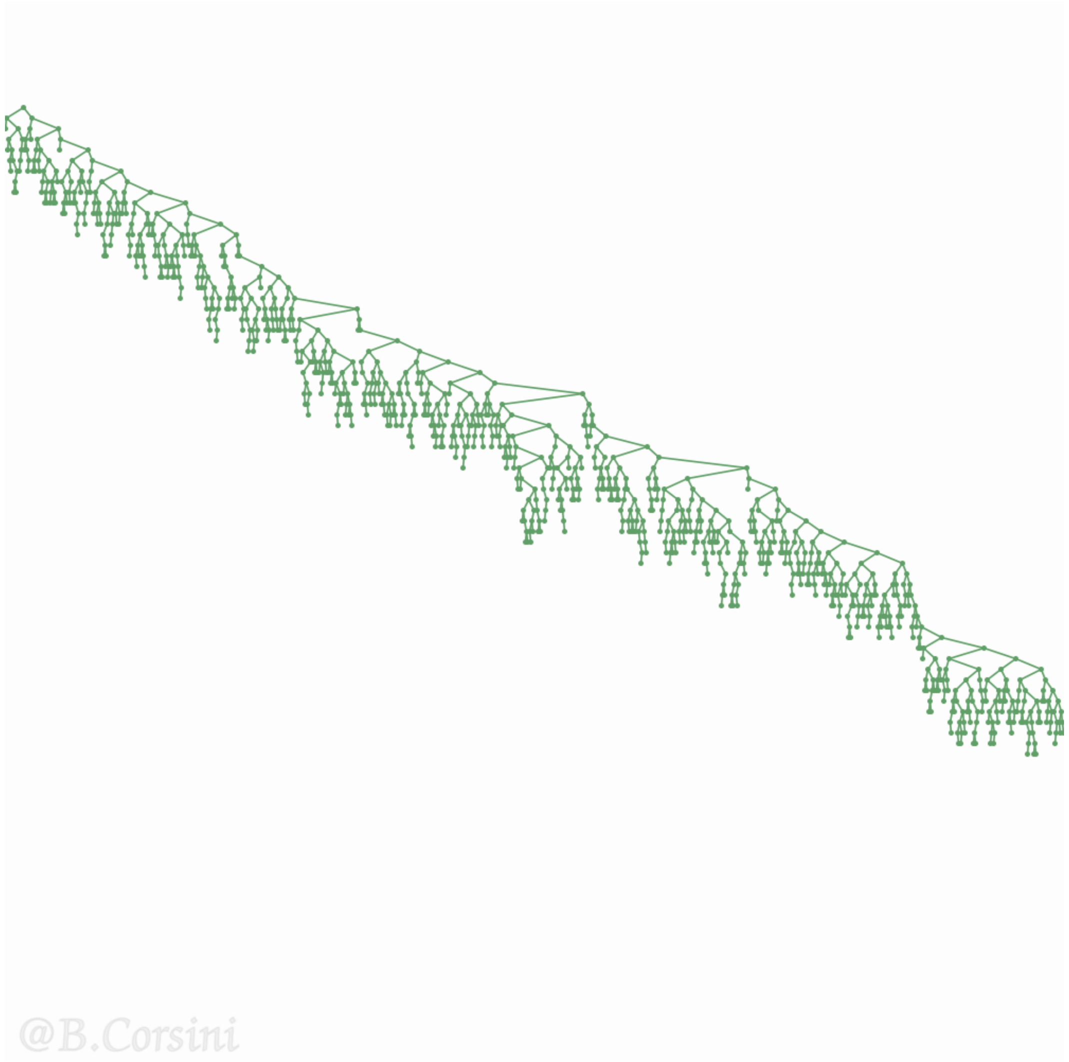

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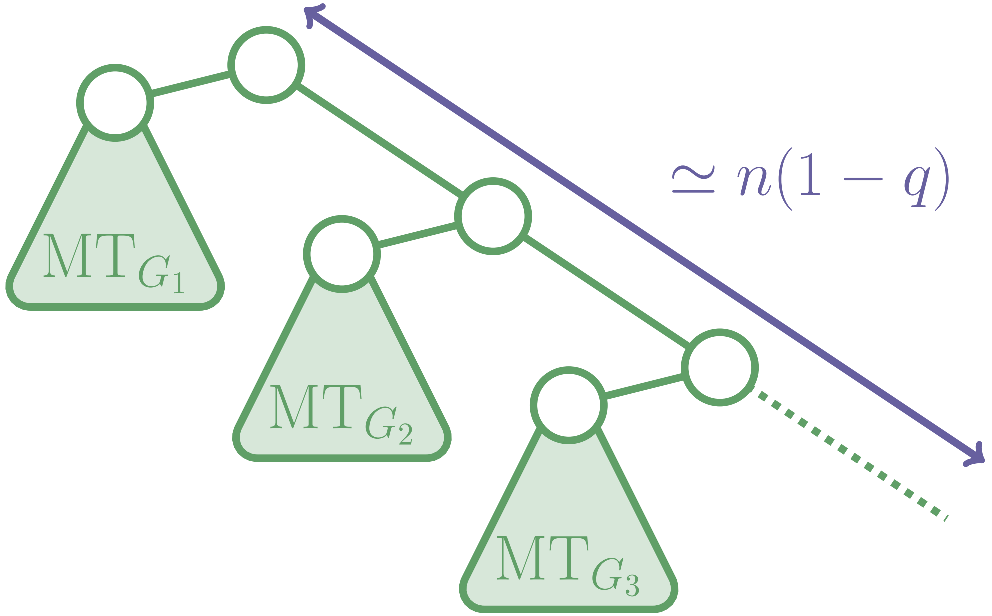

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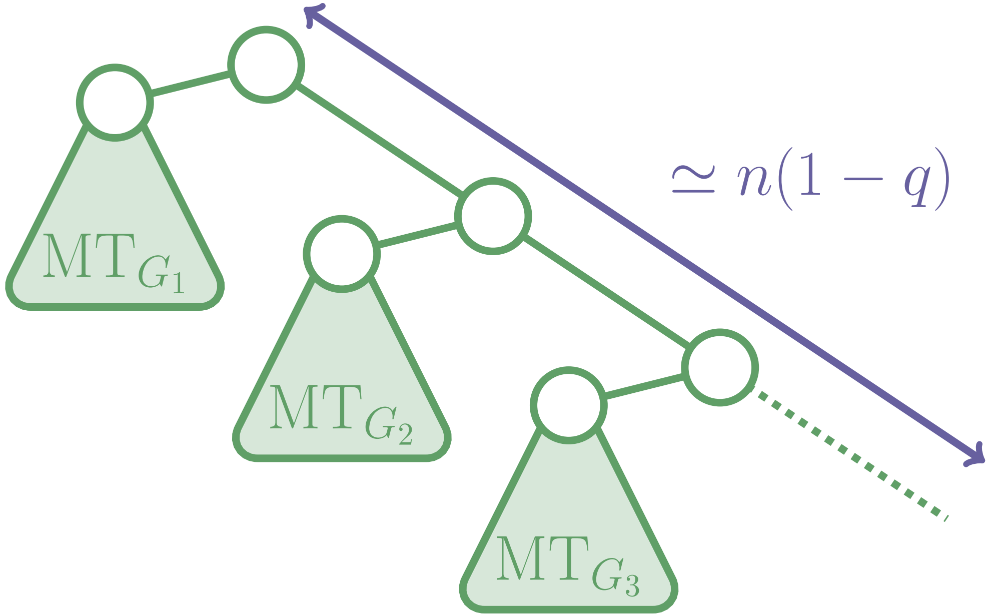

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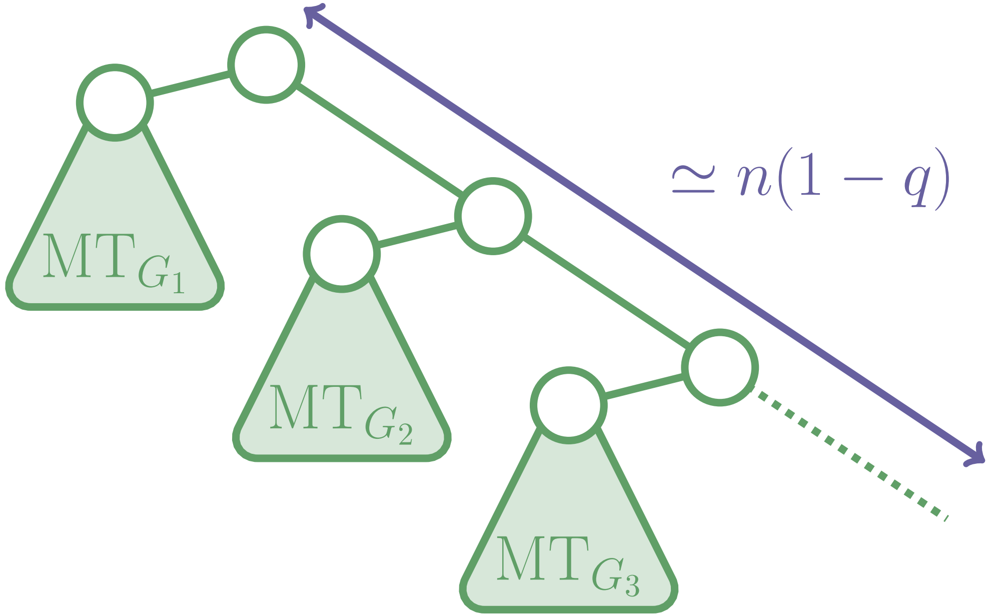

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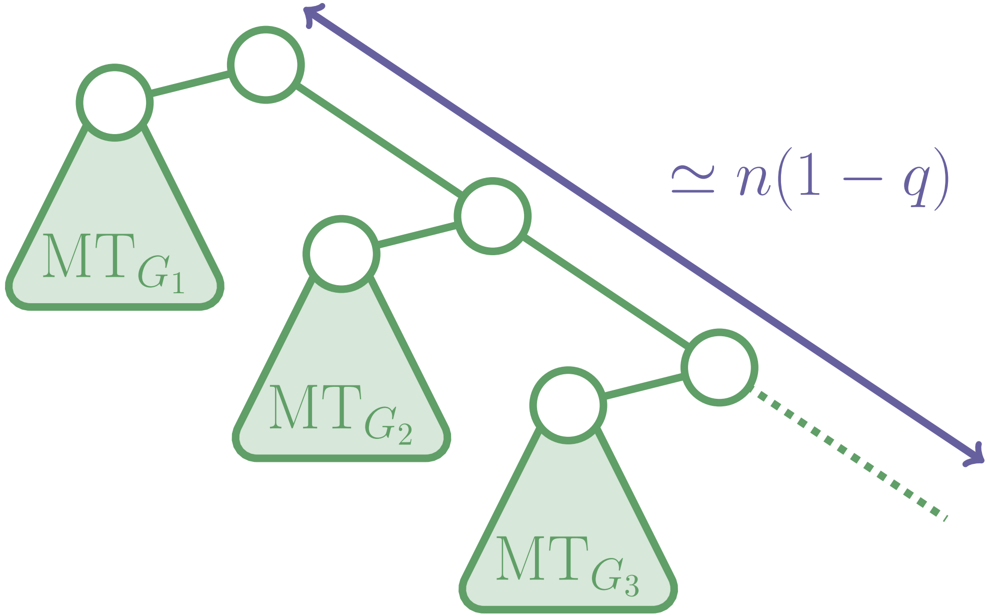

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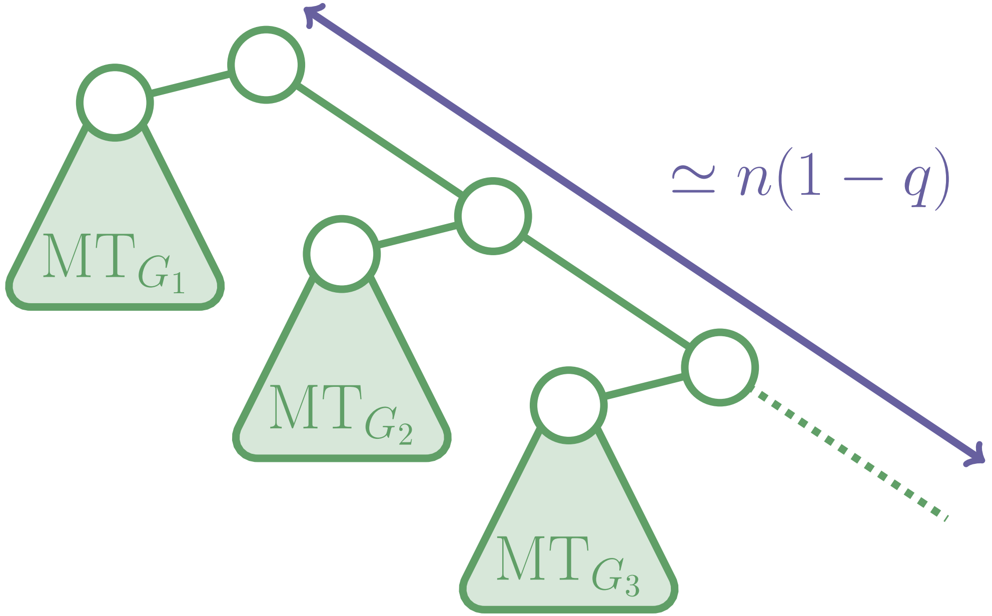

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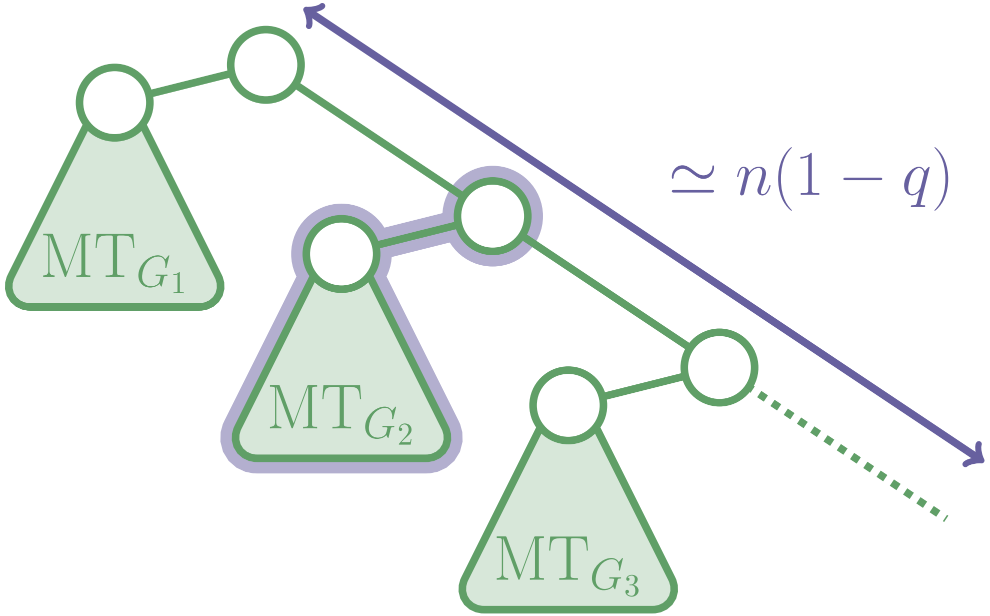

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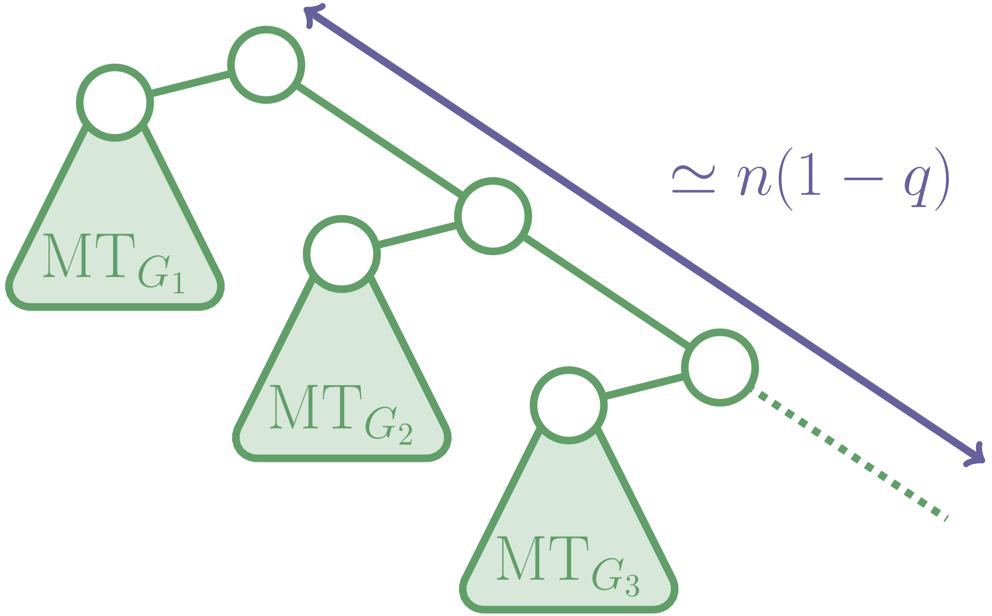

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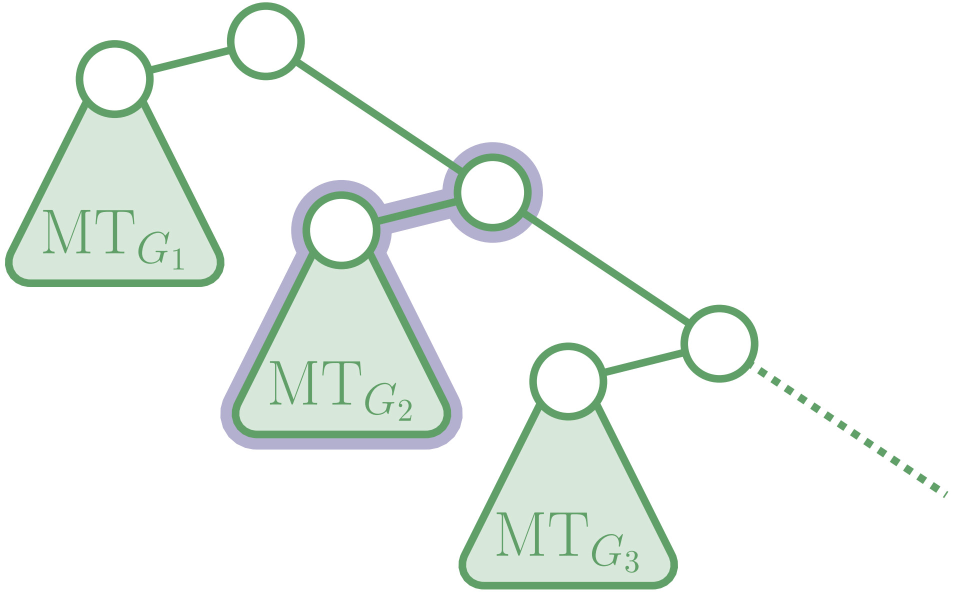

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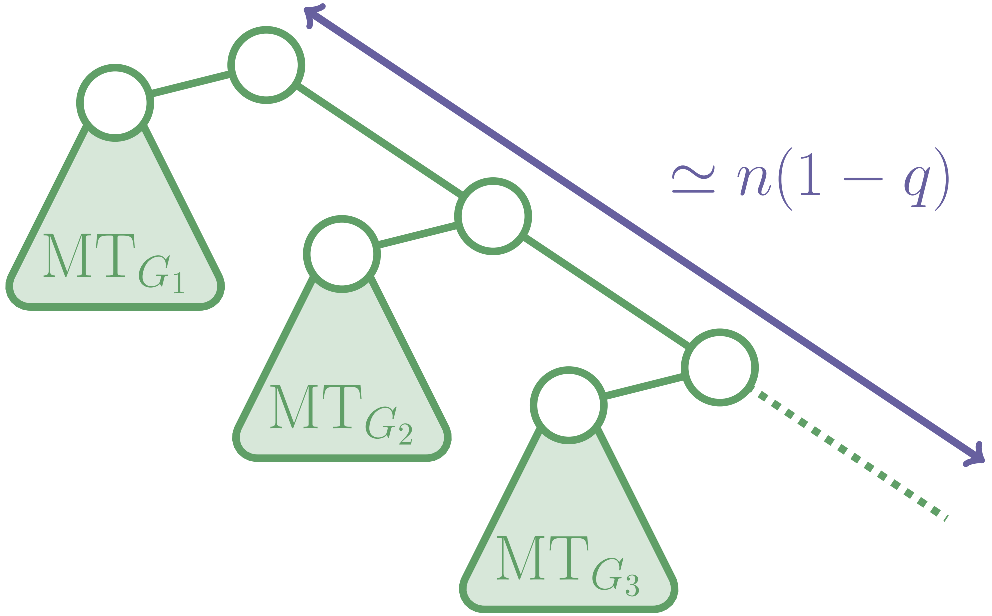

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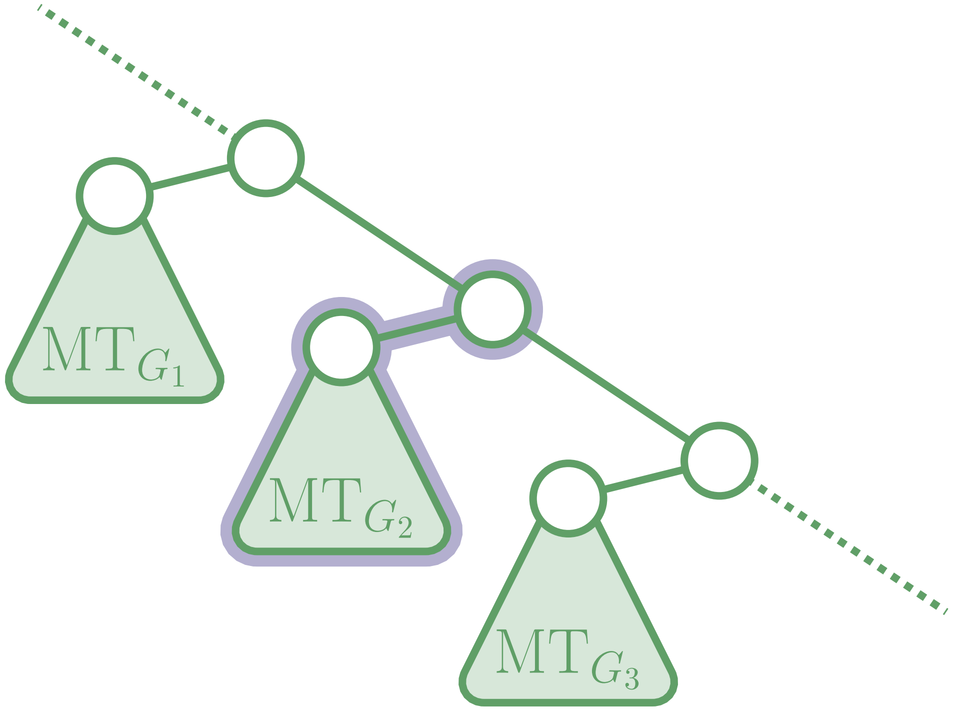

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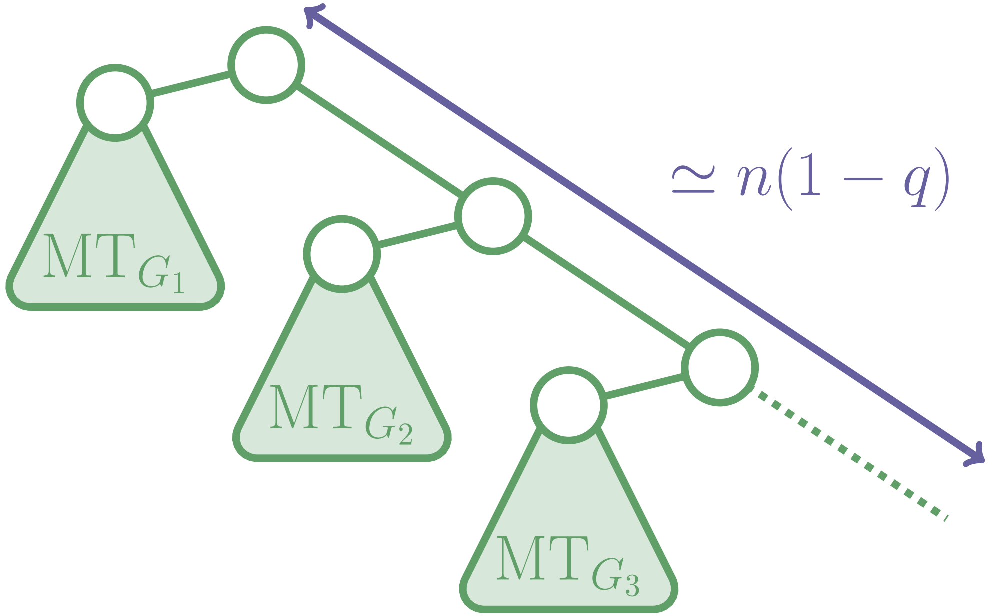

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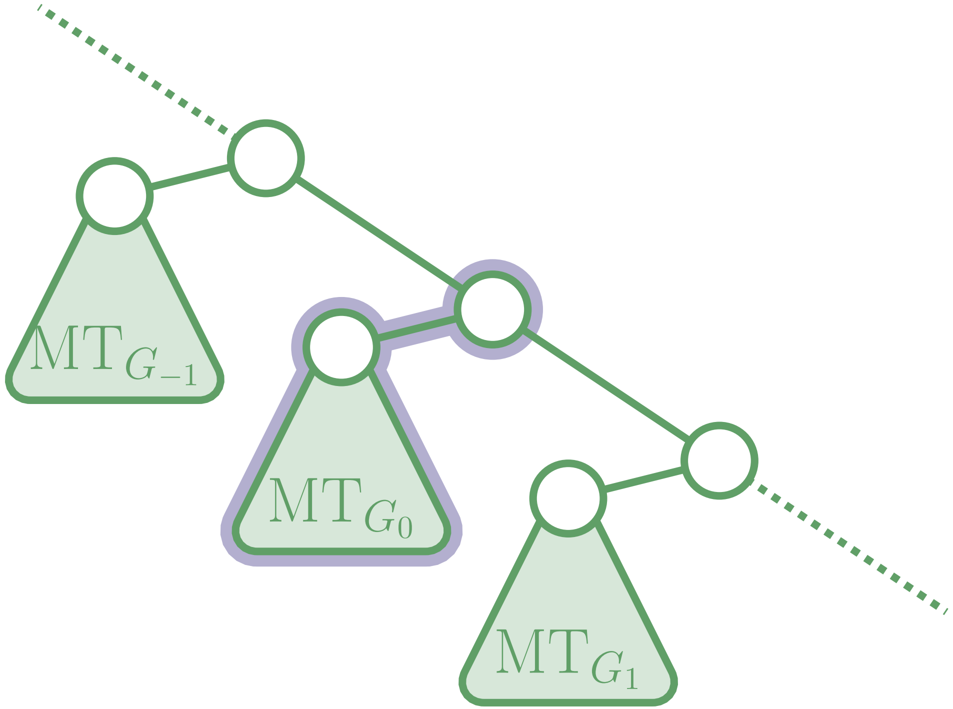

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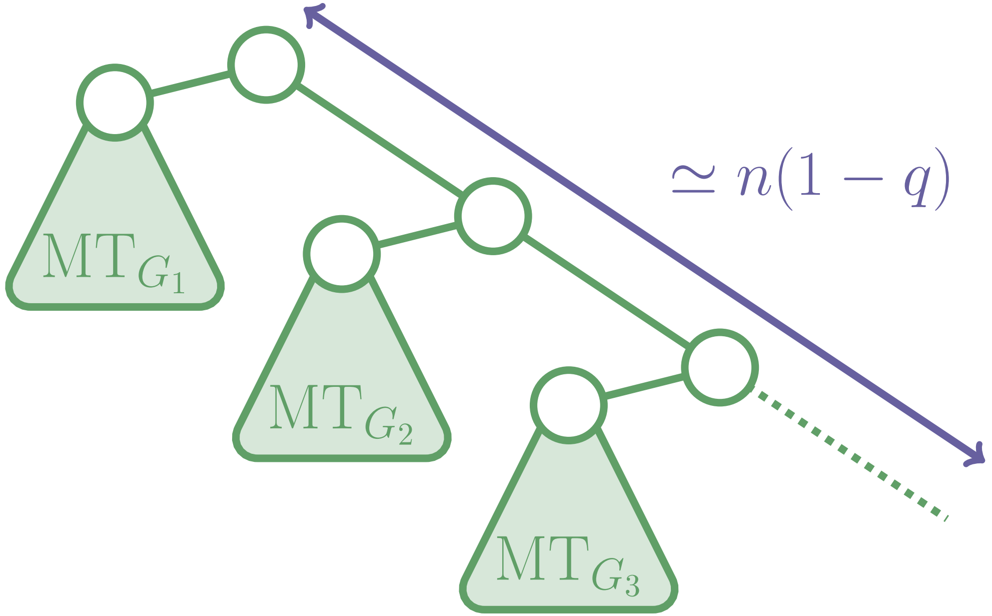

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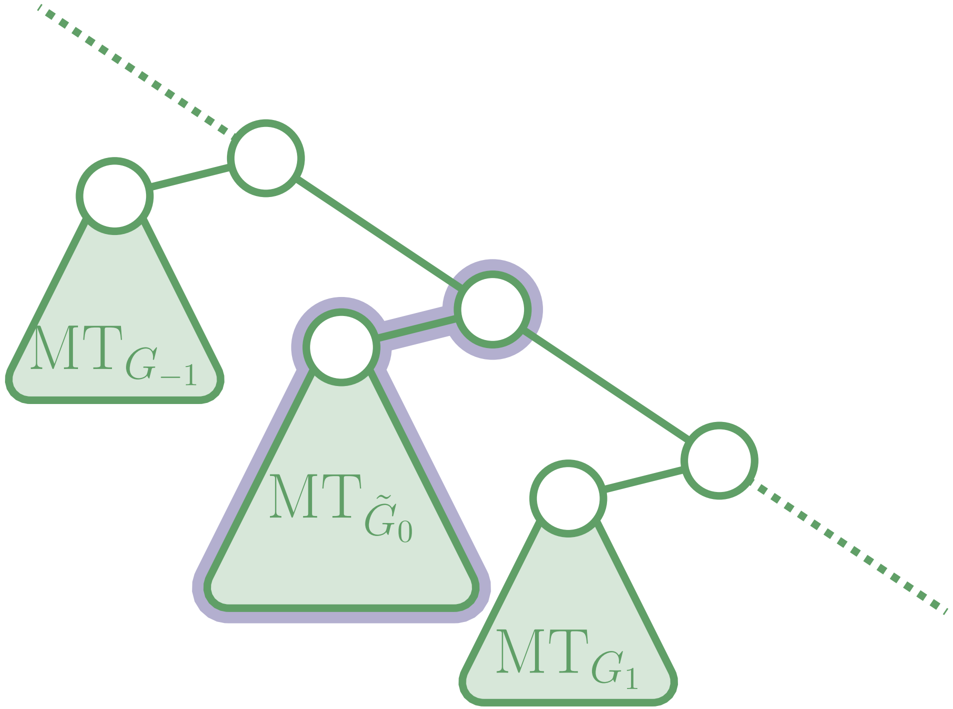

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- Choose o uniformly over $\{0\} \cup V(T_0)$.

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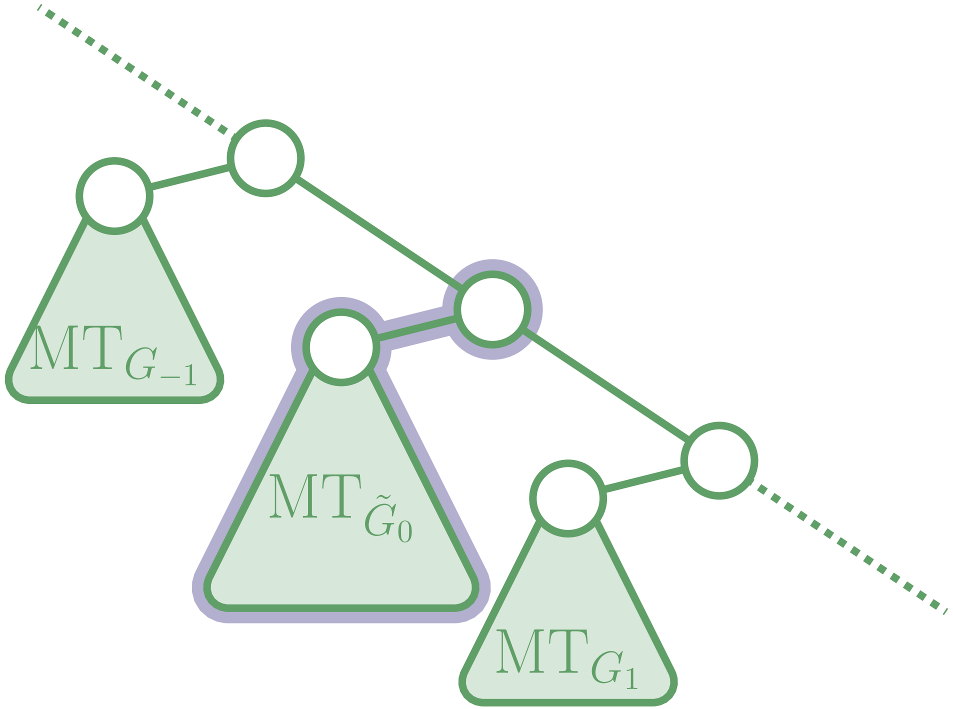

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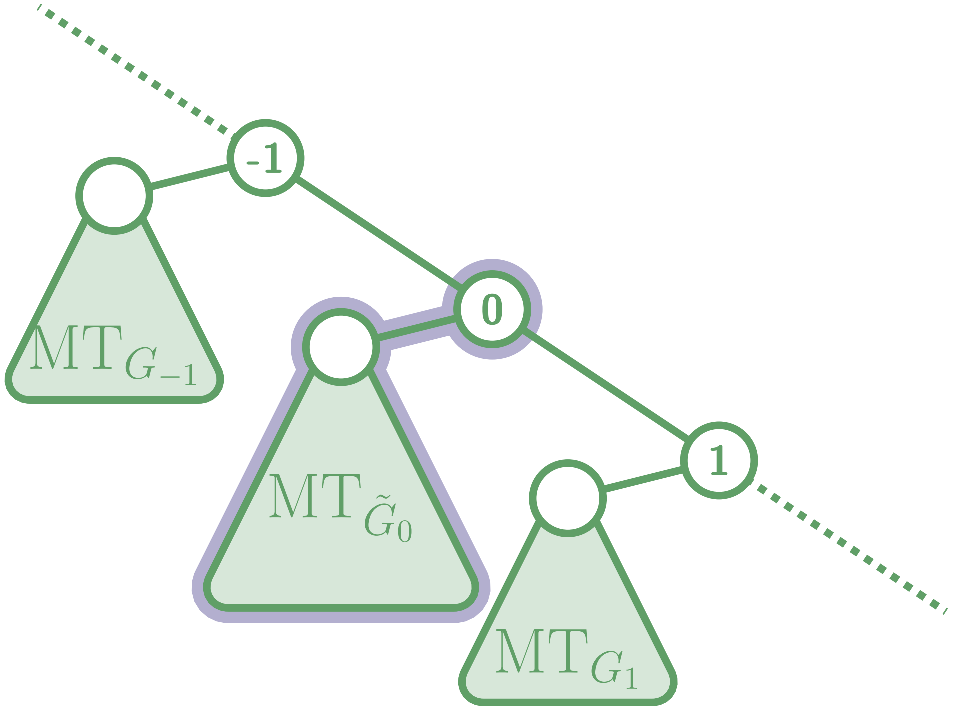

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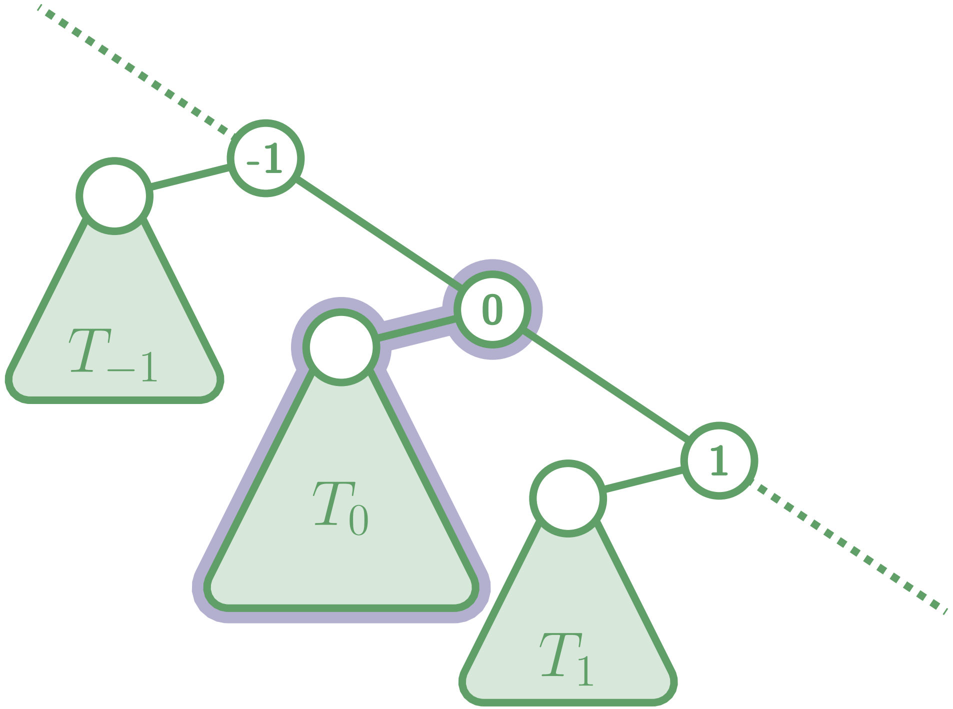

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→ Let us try some cases to understand how they work.

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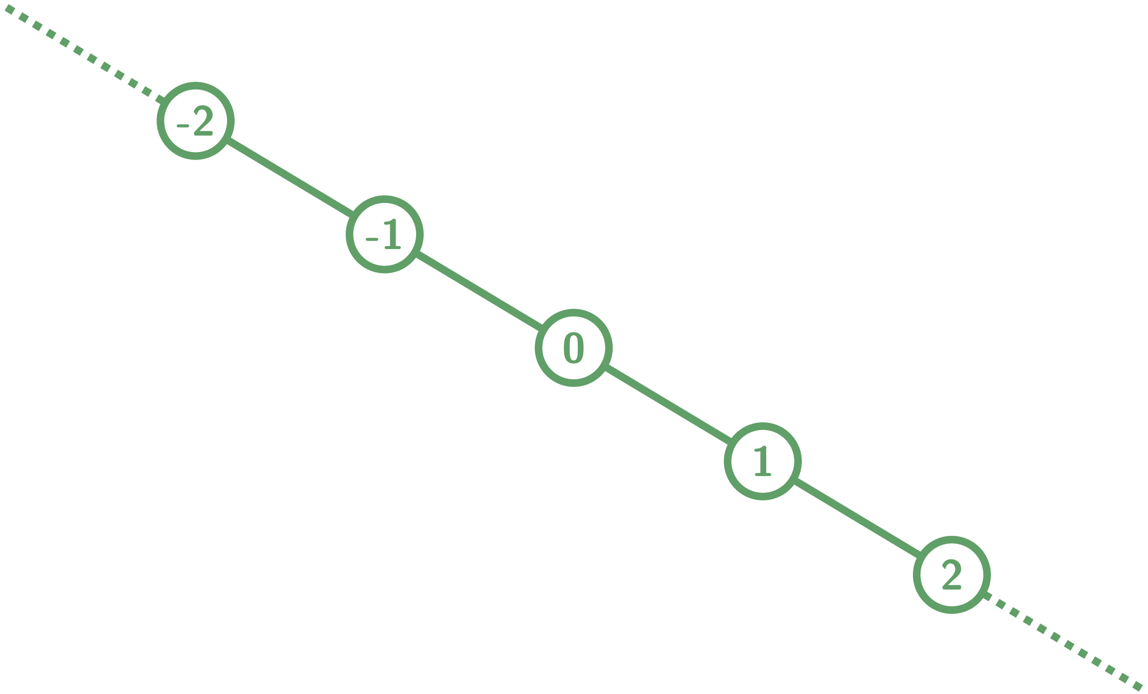

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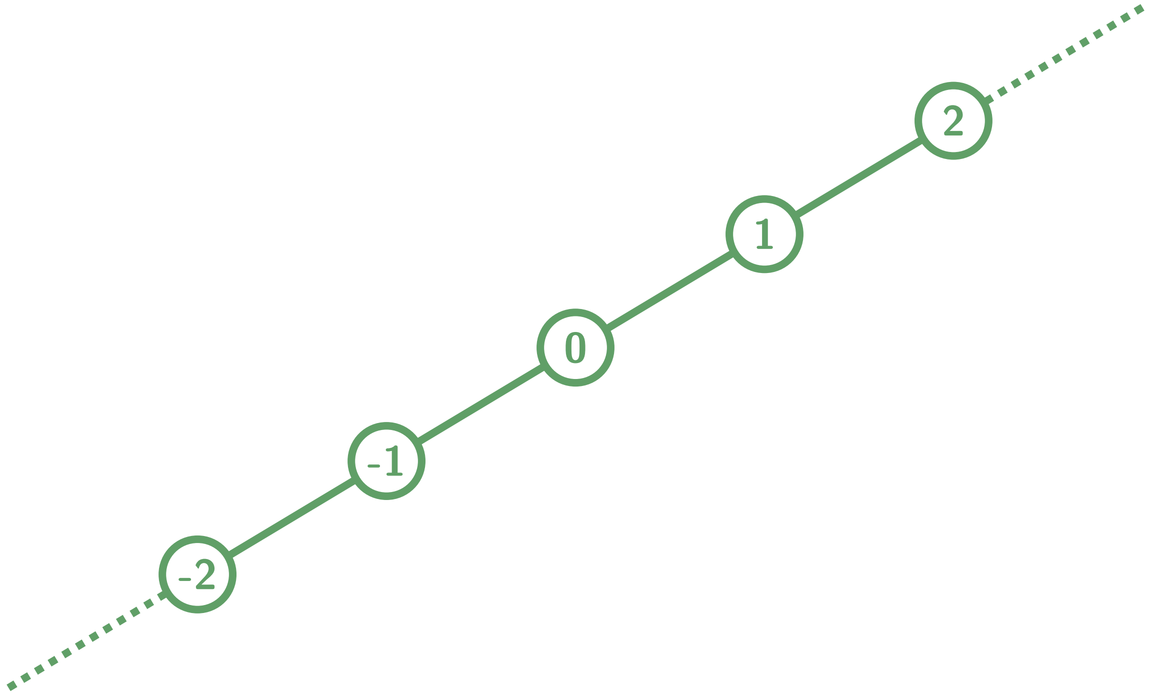

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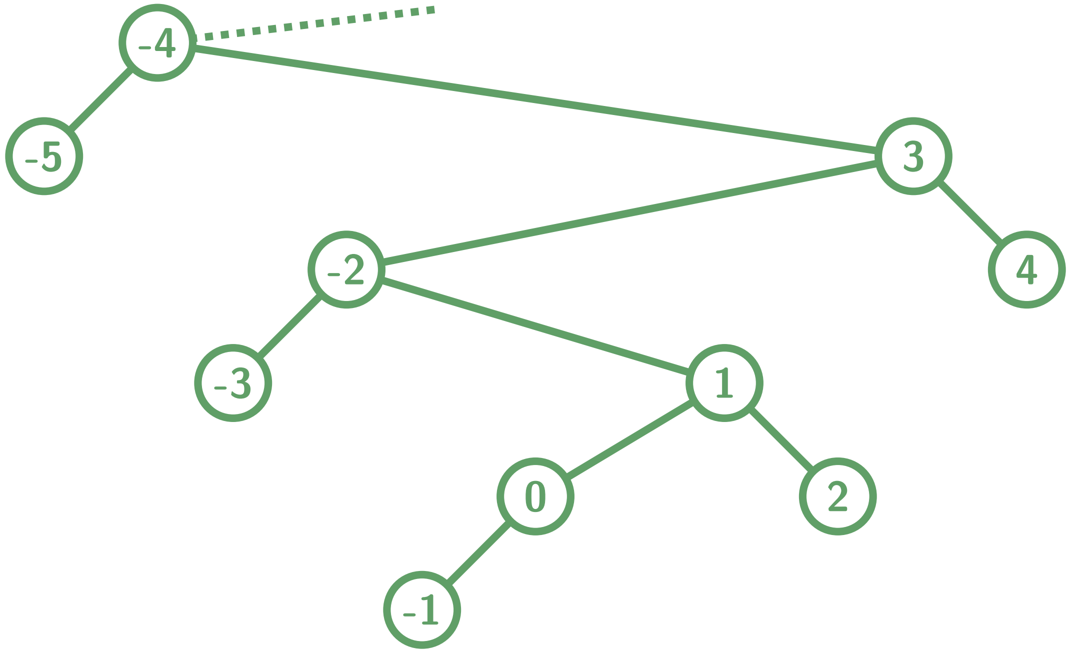

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→ For more details, check out the paper or come talk to me. 👍

References

- Addario-Berry, L., & Corsini, B. (2021). **The height of Mallows trees.** *The Annals of Probability, 49(5), 2220-2271.*
- Corsini, B. (2023). **Limits of Mallows trees.** *arXiv preprint arXiv:2312.13817.*
- Evans, S. N., Grübel, R., & Wakolbinger, A. (2012). **Trickle-down processes and their boundaries.** *Electron. J. Probab, 17(1), 1-58.*
- Gnedin, A., & Olshanski, G. (2012). **The two-sided infinite extension of the Mallows model for random permutations.** *Advances in Applied Mathematics, 48(5), 615-639.*
- Mallows, C. L. (1957). **Non-null ranking models. I.** *Biometrika, 44(1/2), 114-130.*

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